Isogeometric Analysis in G+Smo

A Hands-On Tutorial with Nonlinear Shells

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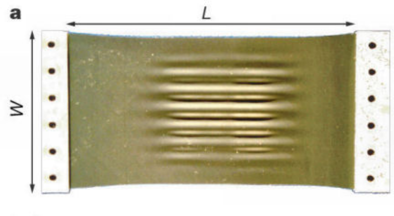
Introduction

Research

- Thin shell analysis
- Isogeometric Analysis (IgA)
- (Post-)buckling methods



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EEMCS/DIAM and 3mE/MTT

Outline

Outline

G+Smo Demo

Features

- Linux, Mac OS, Windows
- Templated C++ library
- Isogeometric Analysis (integration of CAD and analysis)

Geometry + Simulation Modules [1]

gismo.github.io

github.com/gismo

[1] B. Jüttler et al. (2014). "Geometry + Simulation Modules: Implementing Isogeometric Analysis". In: PAMM 14.1, pp. 961–962

Splines

B-spline surface with control points $\mathrm{R} \in \mathbb{R}^3$

$$\mathcal{S}(\xi,\eta) = \sum_{i\in\mathcal{I}}^{N} \psi_i \boldsymbol{R}_i \tag{1}$$

Solution to PDE:

$$\boldsymbol{u} = \sum_{i\in\mathcal{I}}^{N} \psi_i \boldsymbol{\alpha}_i \tag{2}$$

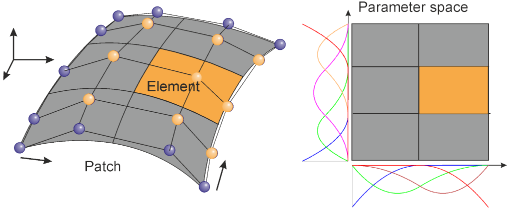


Figure: B-spline description

G+Smo Demo

Isogeometric Analysis

Example: Poisson Equation

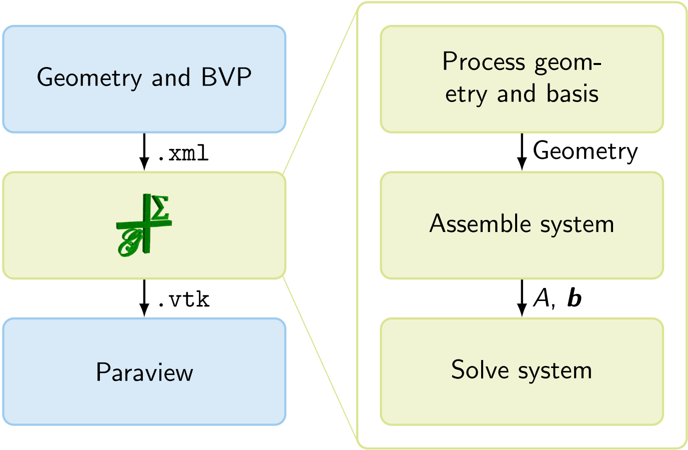
Strong form:

$$\begin{aligned} -\Delta u &= f && \text{in } \Omega \\ u &= u_D && \text{in } \Gamma_D \\ \nabla u \cdot \boldsymbol{n} &= g && \text{in } \Gamma_N \end{aligned}$$

Weak form:

$$\begin{cases} \text{Find } u \in \Sigma_{u_D} \text{ such that} \\ \int_\Omega \nabla u \cdot \nabla \psi \, \mathrm{d}\Omega = \int_\Omega f \psi \, \mathrm{d}\Omega + \int_{\Gamma_N} g \psi \, \mathrm{d}\Gamma \\ \forall \psi \in \Sigma_{u_D} \end{cases}$$

G+Smo Demo

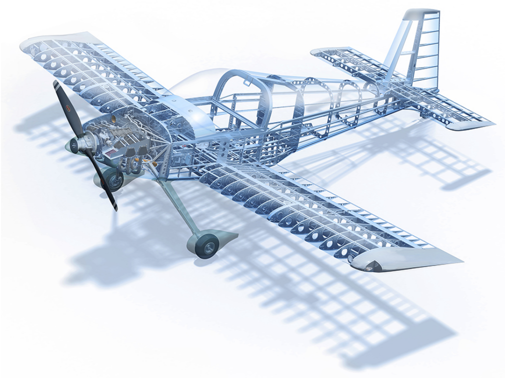


Geometry and BVP
.xml
.vtk
Paraview
Process geometry and basis
Geometry
Assemble system
A, b
Solve system

Outline

The Shell Physics

Thin-Walled Structures



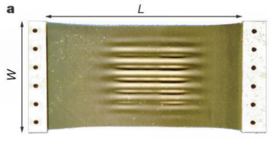
Slender structures; $L \gg t$ $(L/t > 100)$

The Shell Physics

Nonlinearities

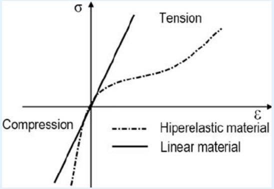
Geometric Nonlinearity

- Large displacements
- Stiffening



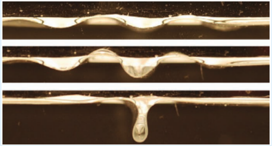
Material Nonlinearity

- Linear material: $\sigma = E\varepsilon$ (Hooke's Law)
- Nonlinear material: $\sigma = C(\varepsilon)$
- e.g. rubbers, plastic deformation



Loading Nonlinearity

- The load 'follows' the deformation
- e.g. (hydrostatic) pressure $\boldsymbol{p} = p\boldsymbol{n}$



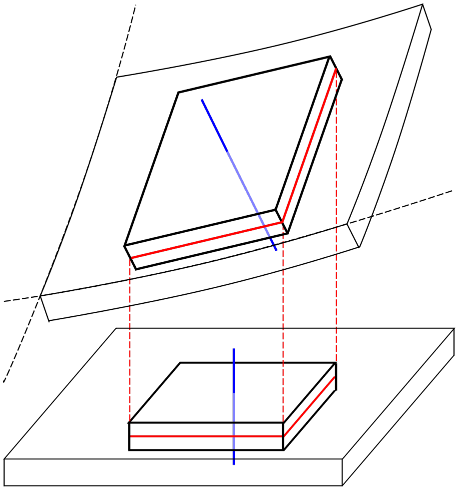
The Shell Physics

Kirchhoff Hypothesis

Definition (Kirchhoff Hypothesis)

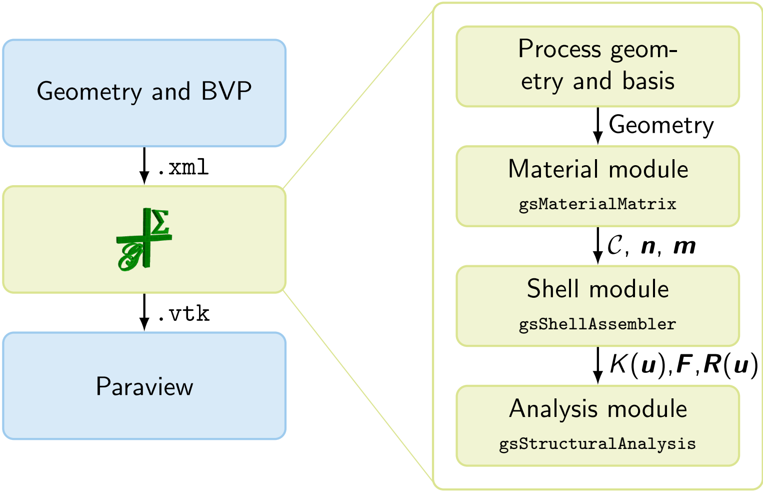
In the Kirchhoff hypothesis, the following is assumed

1. Straight lines perpendicular to the mid-plane before deformation remain perpendicular to the mid-surface after deformation;
2. The normals rotate such that they remain perpendicular to the mid-surface after deformation;
3. The normals do not experience elongation (i.e. they are inextensible).

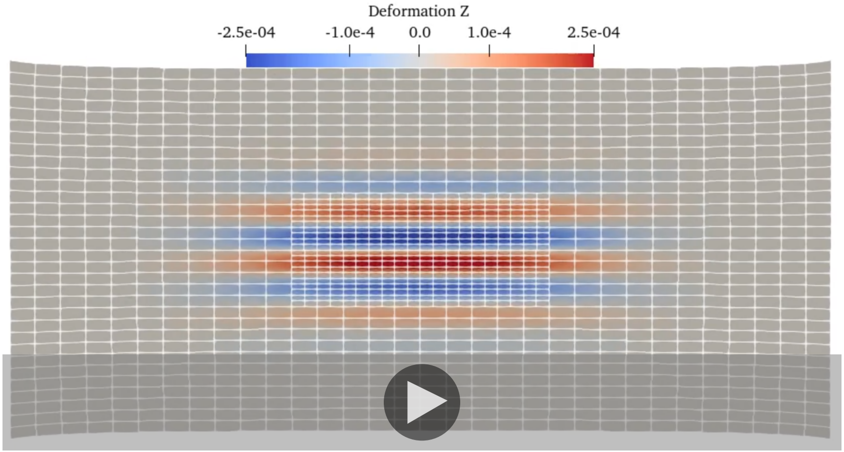


Outline

The Shell Modules



The Shell Modules



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References I

Jüttler, B., U. Langer, A. Mantzaflaris, S. E. Moore, and W. Zulehner (2014). "Geometry + Simulation Modules: Implementing Isogeometric Analysis". In: PAMM 14.1, pp. 961–962.